

Coulomb dissociation of a fast pion into two jets

D.YU. IVANOV^{1,2} L. SZYMANOWSKI^{1,3}

¹*Institut für Theoretische Physik, Universität Regensburg,
D-93040 Regensburg, Germany*

²*Institute of Mathematics, 630090 Novosibirsk, Russia*

³*Soltan Institute for Nuclear Studies, Hoza 69,
00-681 Warsaw, Poland*

Abstract:

We calculate the electromagnetic contribution to the scattering amplitude of pion diffractive dissociation into di-jets which is described by one photon exchange. The result shows that the factorization procedure known for the description of exclusive reactions holds also for this quasi-exclusive process. We find that the longitudinal momentum distribution of di-jets does not depend on the form of the pion distribution amplitude. We discuss the magnitude of the cross section.

1. We study the diffractive dissociation of a pion on a nucleus into two jets. In this process a highly energetic pion interacts with a nucleus and produces two jets (di-jets) which in the lowest approximation are formed from the quark-antiquark ($q\bar{q}$) pair.

$$\pi A \rightarrow q\bar{q}A \quad (1)$$

Early studies of this process can be found in Ref. [1]. Recently it was measured in the E791 experiment [4] and was the subject of further theoretical investigations [2, 3]. The process is of interest for its potential to measure directly the pion distribution amplitude (i.e. the probability amplitude to find in a pion a valence $q\bar{q}$ Fock state with a certain momentum fraction) and to study the effects of colour transparency.

The main contribution to the diffractive process (1) is due to Pomeron exchange, or in QCD language, by the colour singlet gluon ladder. However, this process can also occur as result of the electromagnetic interactions between pion and target nucleus (Coulomb exchange). The strenght of the electromagnetic coupling is αZ (α is the electromagnetic fine coupling constant). For heavy nuclei αZ is not small and therefore one can ask about the size of the Coulomb contribution to (1). The answer to this question is the subject of the present paper.

2. The cross section for this process is largest when the momentum transfer to the nucleus is smaller than the scale of the nucleus form factor $\Lambda \sim 60$ MeV (for Platinum, $Z = 78$). Therefore we consider the process (1) in the case that large transverse momenta of the quark jet ($q_{1\perp}$) and of the antiquark one ($q_{2\perp}$) balance each other, $|q_{1\perp} + q_{2\perp}| \ll |q_{1\perp}|, |q_{2\perp}|$. The diagrams describing the Coulomb contribution to the process (1) are shown in Fig. 1. The wavy line denotes photon exchanged in the t-channel. The large transverse momenta of di-jets result from the hard gluon exchange denoted by the dashed line. They supply a hard scale to the process and therefore we neglect the pion mass. The incoming pion has momentum p_1 , $p_1^2 = 0$. The nucleus mass is M , its momenta in the final and in the initial states are p_2 and p_3 , respectively. The large cms energy squared of the process is given by the Mandelstam variable $s = (p_1 + p_2)^2$, whereas the small momentum transfer squared equal to the photon virtuality k^2 is denoted as $t = k^2 = (p_2 - p_3)^2$.

Let us also introduce the auxiliary Sudakov vector p'_2 such that $(p'_2)^2 = 0$

$$p_2 = p'_2 + \frac{M^2}{\bar{s}} p_1, \quad (2)$$

where $\bar{s} = 2(p_1 p'_2) = s - M^2$. Then, the Sudakov decompositions of the on-shell $q\bar{q}$ momenta in the di-jets are

$$q_1 = z p_1 + \frac{q_{1\perp}^2}{z \bar{s}} p'_2 + q_{1\perp} \quad \text{and} \quad q_2 = \bar{z} p_1 + \frac{q_{2\perp}^2}{\bar{z} \bar{s}} p'_2 + q_{2\perp}. \quad (3)$$

Here, the variable z describes the fraction of pion momentum carried by quark, $0 \leq z \leq 1$. For the corresponding fraction of the antiquark momentum we use the shorthand notation $\bar{z} = 1 - z$. The decomposition of the photon momentum k reads

$$k = \alpha p_1 + \beta p'_2 + k_\perp, \quad (4)$$

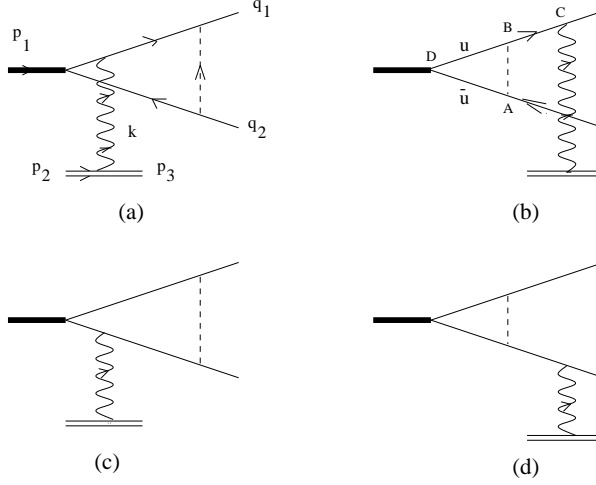


Figure 1: Coulomb contribution to pion dissociation into two jets.

its parameters α, β and the transverse momentum k_\perp are expressed through jets variables

$$\beta = \frac{M_{2j}^2 + (q_{1\perp} + q_{2\perp})^2}{\bar{s}}, \quad \alpha = -\frac{(q_{1\perp} + q_{2\perp})^2 + \frac{M_{2j}^2 \beta}{\bar{s}}}{\bar{s}}, \quad k_\perp = q_{1\perp} + q_{2\perp}, \quad (5)$$

here M_{2j}^2 is the invariant mass of the di-jets

$$M_{2j}^2 = \frac{q_{1\perp}^2}{z} + \frac{q_{2\perp}^2}{\bar{z}} - (q_{1\perp} + q_{2\perp})^2. \quad (6)$$

As already mentioned the main contribution to the process comes from the region of small momentum transfer (or photon virtuality k^2) $t = k^2 = -k_\perp^2 + \alpha\beta\bar{s}$. There $k_\perp \ll q_{1\perp}, q_{2\perp}$, which means that $q_{2\perp} \approx -q_{1\perp}$ and

$$M_{2j}^2 = \frac{q_{1\perp}^2}{z\bar{z}}. \quad (7)$$

The momentum transfer t equals $t = k^2 \approx -(k_\perp^2 + k_{min}^2)$, and according to Eq.(5) its minimal value is

$$k_{min}^2 = \frac{M^2 M_{2j}^4}{\bar{s}^2} = \frac{M_{2j}^4}{4E^2}, \quad (8)$$

the last equality being valid in the rest frame of the nucleus in which the pion energy equals E .

3. Our aim is to calculate the leading asymptotics of the scattering amplitude M in powers of $1/q_{1\perp}^2$ (at the leading twist level). It is given as the convolution of the hard scattering amplitude $T_H(u, \mu_F^2)$ with the pion light-cone distribution amplitude $\phi_\pi(u, \mu_F^2)$

$$M = \int_0^1 du \phi_\pi(u, \mu_F^2) T_H(u, \mu_F^2) \quad (9)$$

where the hard scattering amplitude $T_H(u, \mu_F^2)$ describes the production of a free $q\bar{q}$ pair (the di-jets) in collision of the t -channel photon with q and \bar{q} (the later having momenta

up_1 and $\bar{u}p_1$, respectively), collinear to the pion momentum p_1 . The factorization scale μ_F is of the order of the di-jets transverse momentum $q_{1\perp}$. At lowest order in α_s , which we consider in this paper, $T_H(u, \mu_F^2)$ does not depend on μ_F .

Eq.(9) describes the factorization procedure in QCD which disentangles the contributions to M coming from large and small distances. The soft part is described by the pion distribution amplitude [5, 6], which is defined as follows

$$\langle 0 | \bar{d}(x) \gamma_\mu \gamma_5 u(-x) | \pi^+(p) \rangle_{x^2 \rightarrow 0} = i p_\mu f_\pi \int_0^1 du e^{i(2u-1)(xp)} \phi_\pi(u) . \quad (10)$$

The constant $f_\pi = 131 \text{ MeV}$ is known experimentally from $\pi \rightarrow \mu \nu$ decay, $\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(0) | \pi^+(p) \rangle = i p_\mu f_\pi$.

The hard part, i.e. the amplitude $T_H(u)$ in Eq.(9), is calculable in the perturbative QCD, it is given by four tree diagrams shown in Fig. 1.

The factorization procedure described by Eq. (9) is similar to that used for the hard exclusive processes [5, 6]. Its validity in the case of our quasi-exclusive process can be clarified as follows. Let us consider, for example, the diagram Fig.1(b). The large transverse momentum of quark jets flows along the lines $A - B - C$. Therefore their virtualities are much larger than those of the other quark lines $D - A$ and $D - B$. Thus, at leading twist, the quark lines $D - A$ and $D - B$ have to be considered as being on mass shell and this part of the diagram can be factorized out of the hard part given by the highly virtual quark and gluon propagators.

Calculating the hard amplitude $T_H(u)$ in the Feynman gauge we perform the usual substitution in the nominator of the photon propagator

$$g^{\mu\nu} \rightarrow 2 \frac{p_1^\mu p_2'^\nu}{\bar{s}}, \quad (11)$$

where $p_2'^\nu$ acts on the upper quark vertex. This brings an inaccuracy $\sim 1/\bar{s}$ which at high energies is very small.

Since the energy of a photon k is small the nucleus can be considered as a "scalar particle" with the electromagnetic formfactor $F_{QED}(k^2)$, so the photon-nucleus vertex leads to the factor (see Eq.(11))

$$(ieZ)(p_2 + p_3)_\mu e^\mu F_{QED}(k^2) = (ieZ)(p_2 + p_3)_\mu \frac{p_1^\mu}{\bar{s}} F_{QED}(k^2) = (ieZ) F_{QED}(k^2) . \quad (12)$$

After calculating convolutions over colour and Dirac indices we obtain the contribution of diagram (b) in the form

$$M_{(b)} = -i \int_0^1 du \phi_\pi(u) \frac{(t^a t^a)_{ij}}{4N_c} \frac{2\bar{s}g^2(e_q eZ) f_\pi}{k^2} \frac{\bar{u}(q_1) \frac{\hat{p}_2'}{\bar{s}} (\hat{q}_1 - \hat{k}) \gamma_\mu \gamma_5 \hat{p}_1 \gamma_\mu v(q_2)}{(q_1 - k)^2 (p_1 \bar{u} - q_2)^2} F_{QED}(k^2) . \quad (13)$$

The produced quark and antiquark which have the colours i, j are described by the spinors $\bar{u}(q_1), v(q_2)$. g is the strong coupling constant. The electric charge of the nucleus we denote as eZ , whereas e_q is the electric charge of the quark.

The contributions of the other diagrams in Fig. 1 can be written down in a similar way. One remark is now in order. As we stressed above the hard scattering amplitude can be calculated in leading twist accuracy by considering the pion splitting into on-shell quarks. Then, the sum of diagrams Fig.1(a) and Fig.1(b) as well as the sum of diagrams Fig.1(c) and Fig.1(d) are separately gauge invariant, i.e. both sums vanish when \hat{p}'_2 in the photon vertex is substituted by \hat{k} . This property together with Eqs. (4,5) permits us to substitute in all quark-photon vertices

$$\hat{p}'_2 \rightarrow -\frac{\bar{s}}{M_{2j}^2 - t} \hat{k}_\perp, \quad (14)$$

since the term proportional to p_1 in Eq.(4) gives in our kinematics a contribution suppressed by power of \bar{s} . The substitution (14), being the consequence of gauge invariance, implies that the coupling of the photon to the $q\bar{q}$ pair vanishes linearly when $k_\perp \rightarrow 0$.

The final result for the scattering amplitude corresponding to the sum of diagrams shown in Fig. 1 is given by the formula

$$M^{\pi^+ + A \rightarrow 2j + A} = -i\delta_{ij} f_\pi \frac{2^6}{3^2} \pi^2 \alpha Z F_{QED}(k^2) \alpha_s(q_{1\perp}) \frac{\bar{s}}{(k_\perp^2 + k_{min}^2)(q_{1\perp}^2)^2} \\ (e_u \bar{z} + e_d z) \bar{u}(q_1) \gamma^5 \left[z \hat{k}_\perp \hat{q}_{1\perp} + \bar{z} \hat{q}_{1\perp} \hat{k}_\perp \right] \frac{\hat{p}'_2}{\bar{s}} v(q_2) \int_0^1 du \frac{\phi_\pi(u)}{u}. \quad (15)$$

where $\alpha_s = \frac{g^2}{4\pi}$, and we used the symmetry property $\phi_\pi(u) = \phi_\pi(\bar{u})$. The behaviour of the pion distribution amplitude at the end-points is known: $\phi_\pi(u) \sim u$ for $u \rightarrow 0$, and $\phi_\pi(u) \sim \bar{u}$ for $\bar{u} \rightarrow 0$ [5, 6]. Thus the integral over the momentum fraction u is well defined what confirms that factorization holds for the process we discuss. Eq. (15) is the main result of our paper.

Let us emphasize that the integral of $\phi_\pi(u)$ over u in Eq. (15) generates only an overall factor. Therefore the dependence of the amplitude M on z is universal, i.e. it doesn't depend on the shape of the pion distribution amplitude. It is interesting also to note that the amplitude (15) vanishes for $z = e_u/(e_u - e_d) = 2/3$. Due to the opposite signs of the electric charges of the pion constituents, $e_u = 2e/3, e_d = -e/3$, the contribution of the diagrams (a) and (b) in Fig. 1 cancels the one of diagrams (c) and (d).

The expression in the square bracket on the r.h.s. of Eq.(15) can be put in the form

$$\bar{u}(q_1) \gamma^5 [\dots] \frac{\hat{p}'_2}{\bar{s}} v(q_2) = \sqrt{u\bar{u}} \chi_q [(k_\perp q_{1\perp}) + i([k_\perp \times q_{1\perp}] \vec{\sigma})(z - \bar{z})] \chi_{\bar{q}} \quad (16)$$

where $\chi_q, \chi_{\bar{q}}$ are two dimensional spinors of quark and antiquark. Since the incoming pion fluctuates into a $q\bar{q}$ states with total helicity zero we have in principle to consider two amplitudes having different spin configurations of quark and antiquark as denoted by $(\uparrow\downarrow)$ and $(\downarrow\uparrow)$. However, according to Eq.(16) $M(\uparrow\downarrow) = M(\downarrow\uparrow)^*$, so effectively we deal here with only one independent helicity amplitude.

The Colomb contribution to the process (1) was considered in Ref. [3]. Our result (15) differs from the formula (78) of [3]. According to the factorization formula (9)

we express our result in terms of the pion distribution amplitude $\phi_\pi(u)$. We predict the universal z dependence of the scattering amplitude M , independent of the shape of $\phi_\pi(u)$, moreover M vanishes for $z = 2/3$. Whereas according to Eq. (78) of [3] the amplitude is proportional to the lowest pion Fock state wave function, $M \sim \Psi_\pi(z, q_{1\perp})$ (in our notation). This disagreement is, in our opinion, related to the treatment of the hard gluon exchange in [3]: it is not included in the hard scattering block as is usually done within the QCD factorization approach but it is attributed to the high transverse momentum tail of $\Psi_\pi(z, q_{1\perp})$.

4. Next, we estimate the cross section. For simplicity we use the asymptotic pion distribution amplitude $\phi_\pi = 6u\bar{u}$. Using the standard rules and performing the sum over the colours and over the helicities of produced quarks we obtain

$$(q_{1\perp}^2)^4 \frac{d\sigma^{\pi^+ + A \rightarrow 2j + A}}{d^2 k_\perp d^2 q_{1\perp} dz} = f_\pi^2 \frac{2^5}{3\pi} \alpha^2 \alpha_s^2(q_{1\perp}) Z^2 (e_u \bar{z} + e_d z)^2 \frac{1}{(k_\perp^2 + k_{min}^2)^2} F_{QED}^2(k_\perp^2) [(z - \bar{z})^2 (q_{1\perp}^2 k_\perp^2 - (q_{1\perp} k_\perp)^2) + (q_{1\perp} k_\perp)^2] . \quad (17)$$

For the electromagnetic formfactor of a nucleus (Platinum, $Z = 78$) we use a naive dipole approximation

$$F_{QED}(k^2) = \frac{\Lambda^2}{\Lambda^2 + k^2}, \quad \Lambda = 60 \text{ MeV} . \quad (18)$$

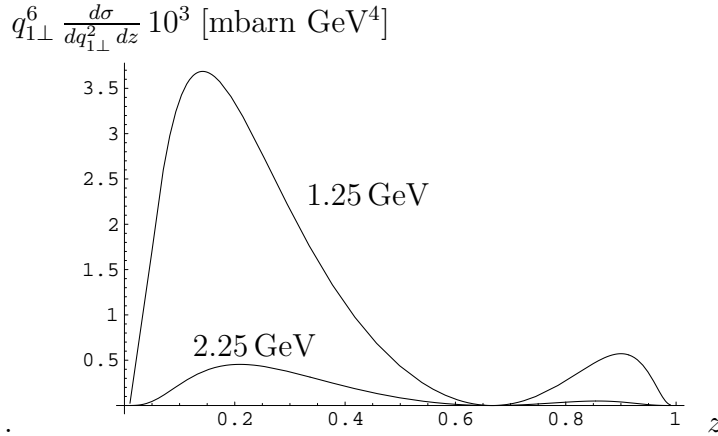


Figure 2: Unsymmetrized cross section for pion dissociation into two jets on Platinum for $q_{1\perp} = 1.25 \text{ GeV}$ and $q_{1\perp} = 2.25 \text{ GeV}$.

In Fig. 2 we present the differential cross section (17) integrated over k_\perp for Platinum and the pion energy $E = 500 \text{ GeV}$. Due to the different charges of u and d quarks the cross section has an asymmetric shape under the interchange $z \leftrightarrow \bar{z}$. As already mentioned it is zero for $z = 2/3$. The cross section vanishes also for $z \rightarrow 0$ and $z \rightarrow 1$, this suppression is caused by the electromagnetic formfactor. According to Eq. (8), in the vicinity of the end-points $z(\bar{z}) \leq q_{1\perp}^2/(2E\Lambda)$, the minimal momentum transfer becomes much larger than the scale of the electromagnetic formfactor Λ .

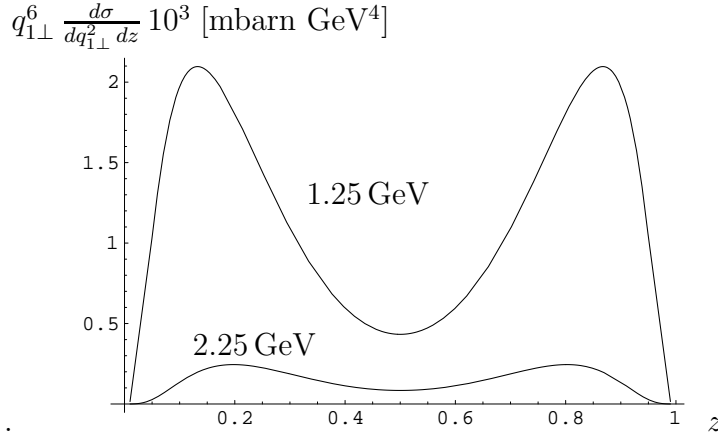


Figure 3: Symmetrized cross section for pion dissociation into two jets on Platinum for $q_{1\perp} = 1.25$ GeV and $q_{1\perp} = 2.25$ GeV.

It is quite difficult to distinguish jets originating from u and d quarks (by identifying the leading hadrons), therefore we show in Fig. 3 the symmetrized cross section, $d\sigma^{sym}(z) = 1/2(d\sigma(z) + d\sigma(\bar{z}))$. The dip structure in the plot for the symmetrized cross section is due to above mentioned zero of the amplitude (15). One could think that the shapes of the plots in Fig. 3 resemble the form of the Chernyak-Zhitnitsky distribution amplitude [6], i.e. that $d\sigma^{sym} \sim (\phi_\pi^{(CZ)}(z))^2$. This similarity is nevertheless accidental since as we explained above the dependence of the amplitude M on z is universal and the shape of curves in Fig. 3 has other reasons.

The Coulomb effect studied in this paper is complementary to the contribution from Pomeron exchange. Although the QCD part of the cross section is large it scales like $1/q_{1\perp}^8$, i.e. it decreases faster at large $q_{1\perp}$ than QED contribution which behaves like $\sim 1/q_{1\perp}^6$. The relative magnitude of the Coulomb part should also be enhanced in the region of small z or \bar{z} . The QCD contribution to the cross section is proportional to $\phi_\pi^2(z) \cdot F_{QED}^2(t)$ which vanishes at $z (\bar{z}) \approx 0$ as $\sim z^2 (\bar{z}^2)$. This behaviour should be compared with the QED contribution given by Eq. (17) which behaves, for $z (\bar{z}) \approx 0$, as $\text{const} \cdot F_{QED}^2(t)$.

The comparison of our results on the Coulomb diffractive pion dissociation into two jets with theoretical predictions or with experimental data is difficult. On the one hand side the theoretical results existing in the literature [2, 3] for the QCD contribution are controversial. In a forthcoming paper [7] we try to solve this issue within the QCD factorization scheme. On the other hand, in the kinematical region covered by E791 experiment the Coulomb effect is difficult to observe due to the large Pomeron background. Also, one cannot compare our predictions for Coulomb contribution directly with E791 data since their absolute normalization is still not known.

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